INVESTIGATION OF THE POSSIBILITY OF USING

A NONHOMOGENEOUS MAGNETIC FIELD FOR THE

FORMATION OF HIGH-DENSITY ELECTRON BEAMS

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A NONHOMOGENEOUS MAGNETIC FIELD FOR THE

FORMATION OF HIGH-DENSITY ELECTRON BEAMS .

Given are experimental data on the influence of the
magnetic field on the characteristics of the Pierce type
axially symmetrical electron gun with a convergent beam.
The gun is enclosed between magnetic screens of such a
configuration,that the magnetic force lines follow the
calculated electron trajectories. This system permits the
formation of electron beams with a high compression and
sharply outlined boundaries,since at some magnitudes of
the magnetic field the influence of the thermal veloci -
ties of electrons at the cathode is eliminated. The re -
sults of experiments are compared with the previously pro-
posed theory.

Introduction .

In the previous article /1/ a system was investigated for the formation of high-density electron beams , consisting of an electron gun,located in a magnetic field, the force lines of which along the whole gun length are directed along the trajectories of the electrons leaving the cathode with zero initial velocity. In this case the magnetic field acts upon the movement of those electrons of the beam only,which leave the cathode with some thermal velocities. In the usual electrostatic electron guns these thermal electrons,together with the aberrations are one of the basic factors worsening the parameters of the beam and impeding the formation of electron beams with a high compression of its area and a high conductance /2,3/. Location of the whole gun in a correspondingly selected magnetic field gives the possibility to decrease considerably the spreading of the beam,caused by the thermal velocities,and in some places along the path of the beam to exclude it completely. The location of these focusing points ,where the thermal electrons shape in a certain scale a picture of the cathode,can be determined according to formula (14) of article /1/ by the following equality

$$\eta B_{\ast} \int_0^t \left(\frac{r_k}{r_t}\right)^2 dt = 2\pi n, \quad n = 1, 2, 3, \ldots \tag{1}$$

Here $\eta = \frac{e}{m}$ -ratio of the electron charge to its mass; r_k - radius of beam at the cathode; r_e - variable distance of the electron, leaving the edge of the cathode with an initial velocity zero, to the beam axis; t- transit time of this electron from the cathode to the point of the beam being examined ; B_{zk} - magnetic induction on the cathode. Formula (1) was derived for a paraxial electron flow with axial symmetry, basically laminar, with a constant current density in the crossection. For the derivation of formula (1) it was assumed that the magnetic force lines go along the trajectories of the electrons leaving the cathode with a zero initial velocity, and therefore the axial component of magnetic induction B_z varies along the axis of the beam z per

$$B_z = B_{zk} \left(\frac{r_k}{r_e}\right)^2 \qquad (2)$$

where r_e is a function of z .

In connection with the multiplicity and seriousness of the assumptions made for the development of the thermal electron trajectory equation, and the determination of the current density distribution in the article /1/, an experimental examination of the derived formulas is desirable. In the work /4/ the beam diameter on the anode of a Pierce gun with parallel flow (special case $r_e \equiv r_k$ in condition (1)), located in a uniform magnetic field,

was measured. It seems that from the practical point of view analogous measurements for a gun with convergent electron beam are more interesting,since this forming system enables us to achieve a higher current density.

The methods and results of an investigation of a Pierce gun with convergent beam,located in a magnetic field,are described below. In this type of electron guns between the cathode and anode /5/ is created the same type electrical field as in the spherical diode,and as a result of this the electrons after leaving the cathode , move along convergent straight lines. After entering through the opening in the anode into the space free from external fields,the beam,originally convergent,reaches its minimum radius r_m ,and then due to its own space charge -starts diverging.

The integral $\int \left(\frac{r_k}{r_a}\right)^2 dt$, ,contained in the expres - sion of the thermal electron trajectory and the condition of focusing(1),is calculated for the Pierce gun in the work of Cutler and Hines /2/. Substituting its value into (1) we obtain

$$r_k B_{,r} \sqrt{\frac{1}{2U_a}} \left\{ (-\alpha)^{3/2} \int\limits_{1}^{\left(\frac{r_k}{r_a}\right)} \frac{d\left(\frac{r_k}{r_a}\right)}{(-\alpha)^{3/2}} + 3 \frac{r_k}{r_m} \sqrt{\frac{\pi}{2}(-\alpha)^2} \left[\Phi\left(\sqrt{\ln\left(\frac{r_a}{r_m}\right)}\right) \pm \right. \right.$$

$$\left. \left. \pm \Phi\left(\sqrt{\ln\left(\frac{r_a}{r_m}\right)}\right) \right] \right\} = 2\pi n, \quad n = 1, 2, 3, \ldots \qquad (3)$$

Here $\overline{r_k}$ and $\overline{r_a}$ -curvature radii of cathode and anode

.4.

in the equivalent spherical diode; r_a -beam radius at the anode; U_a - accelerating voltage at the gun anode ; $(-\alpha)^2$ - Langmuir - Blogett function for spherical diode, dependent on the ratio of radii $\overline{r}_k$ / $\overline{r}_a$ /5/ . Function Φ -integral of probability of errors

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx. \tag{4}$$

The plus sign is taken in formula (3),when examining a point beyond the minimum ($z > 0$),the minus sign - for points between the minimum and anode ($z < 0$).

1. Creation of a Nonhomogeneous Magnetic Field , the Force Lines of which Are Directed Along the Electron Trajectories.

For the type of electron gun being investigated , the beam profile $r_e(z)$ can be easily calculated in case of absence of thermal velocities,and therefore using formula (2) we can determine the required course of the magnetic field along the electron beam axis $B_z(z)$. The corresponding arrangement of magnetic materials and sources of the magnetic field can be determined by the known methods of modeling and approximated calculations.

Evidently the realization of a field described by formula (2) may be impossible in a number of cases,e.g. where sharp bends of electron trajectories occur,as at the anode of the Pierce gun.

We can ,however,assume that small deviations of
magnetic force lines from the exact course do not exert
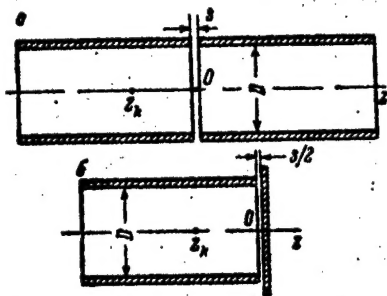a considerable influence on the investigated phenomenon .

Fig. 1.

Taking the above in consi-
deration a simple form of   -
magnetic screens,as shown
on Fig. 1 ,was used for cre-
ating a magnetic field in
the Pierce gun.The screens,
shown on Fig. 1a,are two
cylinders with a diameter D,

made of a material of high magnetic permeability μ and
separated by a narrow gap.

If a magnetic field is produced in the gap by an
outside coil or permanent magnet,the magnitude of this
field in direction of axis z can be expressed with satis-
factory exactness for $\mu = \infty$ by the formula /6/

$$B_z = B_{zm} \frac{1}{\text{ch}^2\left(2.63\frac{z}{D}\right)}. \qquad (5)$$

Here B_{zm} - maximum magnetic field, at the gap
(z = 0) .

The Pierce gun was located between the screens in
such a manner that the minimum beam crossection was exact-
ly at the gap,but the screen diameter D was selected so
that

$$\text{ch}\left(2.63\frac{z_k}{D}\right) = \frac{r_k}{r_m}, \qquad (6)$$

where $|z_k|$ is equal the distance from the cathode to the minimum crossection of the beam,as determined by the gun computations. The force lines of the magnetic field , selected this way,cross the respective calculated trajectories of the nonthermal electrons in two points: at the cathode plane and at the minimum. Although a deviation of the force lines from the trajectories takes place along the beam at other points,it is not great(for a gun with $\bar{r}_k / \bar{r}_a = 2.25$ and $\theta \approx 15°$ it does not exceed 10% of the radius).

For screens per Fig. 1a the plane $z = 0$ may be substituted by a plane magnetic screen,having infinite permeability,and the field structure will be maintained.Therefore the required field distribution for $z \leqslant 0$ can be obtained with screens shaped per Fig. 1b . Such screens can be used in the case when it is required to "stabilize" the electron beam to one side of the minimum only,for example in case of a gun combined with a magnetic focusing system.

The diameter D of the magnetic screen must be made smaller with the increase of compression ,and with shortening the distance between the cathode and the minimum. The last two quantities for the Pierce gun are given by the ratio $\bar{r}_k / \bar{r}_a$ and perveance of the beam $P = I/U_a^{3/2}$. For a high compression and high perveance the calculated

diameter D of the screen may become smaller than the gun electrodes (in particular the cathode diameter $2r_k$), which proves the impossibility of using screens of this shape.Fig. 2 shows the relationship of the perveance P and 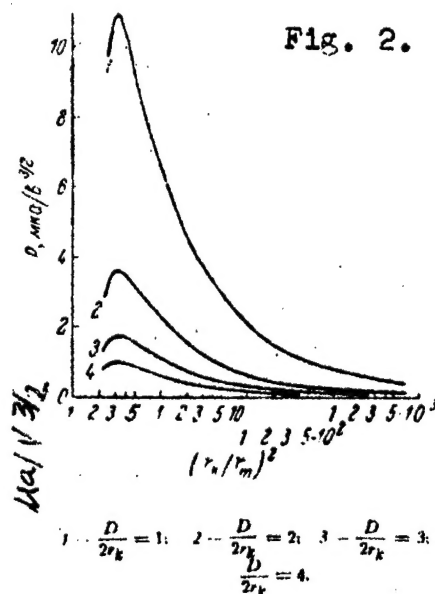compression r_k^2/r_m^2 for different diameter ratios of screen and cathode - $D/2r_k$. The curves of Fig. 2 are derived on the basis of formulas (5) and (6),where z_k was expressed through perveance P with the aid of graphs used for computing the Pierce gun, given in /5/. The curve corresponding to $D/2r_k = 1$ is the limit case,which is practically unattainable. The other curves may be easily achieved.Fig. 2 proves that the requirement of a high beam perveance impedes the achievement of a high compression.

Fig. 2.

$$1 - \frac{D}{2r_k} = 1; \quad 2 - \frac{D}{2r_k} = 2; \quad 3 - \frac{D}{2r_k} = 3; \quad 4 - \frac{D}{2r_k} = 4.$$

It should be mentioned that curves of Fig. 2 are constructed for cylindrical screens only ,with $\mu = \infty$ .For stabilization of beams with parameters higher than those shown on Fig. 2,screens of another shape must obviously be used.
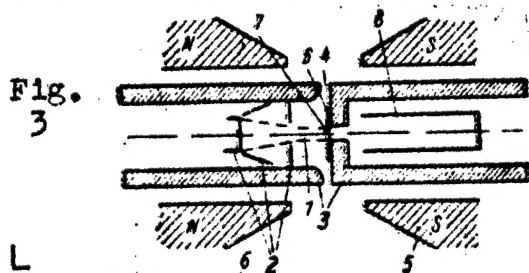
In conducting the experimental part of the work

basically cylindrical screens ,with slight deviations,
which sometimes were of advantage from the construction
point of view,were used. Screens were manufactured of
Armco iron . The magnetic measurements on the screen axis
were made by the ballistic method,using measuring coils
of 4-4.5 diameter and 1 mm length,with o.o2 - o.o3 mm
windings.

The ballistic coils had two windings,one over the
other; the inner,less sensitive coil was used for measur-
ing the magnetic field in the region of the gap,but both
coils connected in series,-for measurings at the cathode.

2. Experimental Check of the Magnetic Field
Influence on the Performance of the Pierce Gun .

Fig. 3 shows the principal scheme of one of the con-
structed apparatuses. The beam 1 ,leaving the Pierce gun
2 ,moves along the whole path from the cathode in the mag-
netic field inside screens 3 ,made of Armco iron. The mag-
netic screens 3 are located between the poles of electro-
magnet 5,and therefore in the gap 4 a magnetic field is

Fig.
3

produced,the force lines of
which go along the nominal
electron trajectories in the
gun. The configuration of

. 9 .

screens in general corresponds to Fig. 1 b .The gap bet -
ween them is located opposite that point of the beam where
the minimum crossection should be ,if there were no therm-
al velocities.

On the beam path,also at the place of minimum rad-
ius,is located a tantalum target 6 ,in the shape of a disc
with a very small (0.07 - 0.08 mm) opening 7 in the cen-
ter. The current passing through the opening and caught by
collector 8 ,is proportional to the current density at
the location of the opening.With variations of current in
the electromagnet the current density at the target must
vary too,reaching its maximum for such magnitudes of the
magnetic field in the gun,which satisfy the focusing con-
dition (3).

The gun investigated in this apparatus had the fol-
lowing parameters: $\bar{r}_k/\bar{r}_a$ =2.25, θ =15^0 , $\left| z_k \right|$ =14.7 mm ,
r_m = 0.38 mm , r_k =2.25 mm,perveance P = o.4.10^{-6} a/v$^{3/2}$.
The calculated compression,ignoring the thermal velocities,
is 34. The gun computation was made according to Pierce
/5/,with a correction of the focusing length of the anode
lens per /7/. The configuration of gun electrodes was
found with the aid of an inclined electrolytic tub /5/.All
parts of the gun,with exception of the oxide coated catho-
de core (nickel),were made of nonmagnetic materials.Each

⌐electrode was fastened with three traverses to the common⌐
ceramic base of the gun.

The complete apparatus,except the electromagnet 5
poles,was enclosed in a glass bulb and tested under va -
. cuum and fused;the high vacuum was maintained by barium   -
gas absorbers.

The measured ratio of the field at the minimum
(B_{zm}) to the field at the cathode fluctuates between 26
to 32 for variations of the field at the cathode from 0
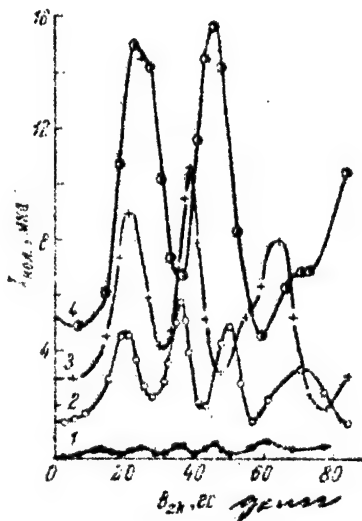to 85 gauss,reaching the maximum for a field of B_{zk}=40gauss.

Fig. 4.

Fig. 4 illustrates the relation -
ship of the collector current I_c
to the magnetic field at the
cathode B_{zK} for different acce-
lerating voltages: 1- 100,2-200,
3- 300,4- 400 v.The curves show ,
as should be anticipated, that
the current density at the measu-
rement point of the beam has seve-
ral,periodically repeating maxima.
The distance between the maxima
increases with increase of the voltage.The current density
at the maxima is several times higher than during absence
of the magnetic field (B_{zk} =0) . This confirms the fact

of thermal electron focusing at some magnitudes of the magnetic field.

The quantitative examination of the focusing condition (3) is of great interest. Substituting the gun parameters into formula (3) we obtain the condition of focusing of thermal electrons in the plane of the minimum beam crossection: $B_{zK}/\sqrt{U_a} = 1.06n$, where B_{zK} is expressed in gauss, U_a - in volts, n- an integer, n = 1,2,3,... corresponding to the "order" of focusing . On Fig. 5 the horizon-
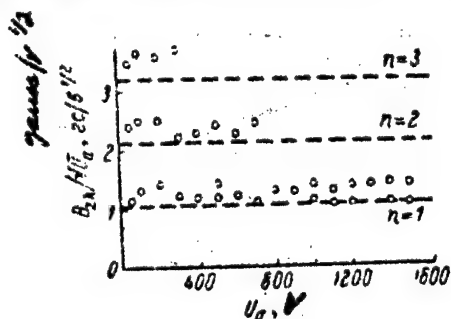

Fig. 5.

tal dotted lines give the calculated relationship $B_{zk}/\sqrt{U_a}$ to U_a, but the points correspond to the maxima of curves I_c (B_{zk}) . The experimental points are located somewhat above the calculated straight lines, that is the focusing requires a somewhat stronger magnetic field than calculated. But this difference can be regarded as small, considering the multiplicity of assumptions made when deriving formula (3), and the inexactness of beam parameter calculations. It should be pointed out , that for higher anode voltages the shape of the curves, analogous to those of Fig. 4, becomes somewhat complicated: e.g. the maxima vertices double slightly.

Therefore a more detailed investigation of the beam, than the one described above, appeared necessary. For this purpose a number of instruments were constructed , permitting determination of current density distribution in different crossections along the beam. Fig. 6 shows the
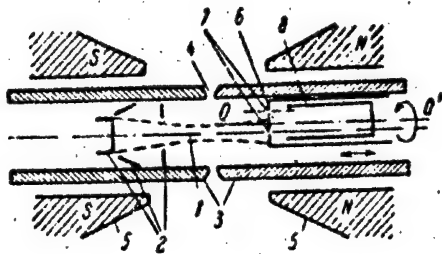

Fig. 6.

section of one such instrument. The beam 1 is generated by an electron gun of the same type as before(see Fig. 3). The gun is located in a magnetic field, produced by cylindrical screens 3 (analogous to Fig. 1 a) , and the minimum beam crosssection is located at the

gap. The screens are located between the poles of an electromagnet ,which is outside of the vacuum bulb.The beam, leaving the gun, hits the tantalum target (6),which can rotate around axis OO',located at some distance off the common axis of the gun and screens. The target has 4 open - ings 7 of a very small diameter (0.06 - 0.08 mm).Each opening is located at a different distance from the axis of rotation.

During rotation of the target the openings pass

through the beam one after the other,and even if the tar-
get and gun are not exactly centered,one of the openings
will pass close to the center of the beam—necessary condi-
tion for a correct measurement of the distribution of cur-
rent density in the beam. The current passing through ope-
ning and captured by collector 8 ,gives the distribution
of current density on the radius. This method permits the
examination of very small diameter axially-symmetrical
beams,where an impossible high exactness of mechanical
assembly would be needed,if the moving target had only
one opening.

The mechanism of the instrument moves the target
simultaneously with rotation (1 mm for each turn),thus
making possible the examination of different beam sections
on a 50 mm length. The movement is transferred into the
bulb by means of an iron armature and an electromagnet ,
rotated by an electromotor.All measurements were made
automatically,and the collector current I_c was registered
on a film by a loop-oscillograph. More detailed data on
this method of measuring current density are given in /8/.

Fig. 7 gives the characteristics of magnetic
screens ,used in the described apparatus. The curve 3 gi-
ves $B_z(z)$ for the current I_m= 0.8 in the electromagnet ,
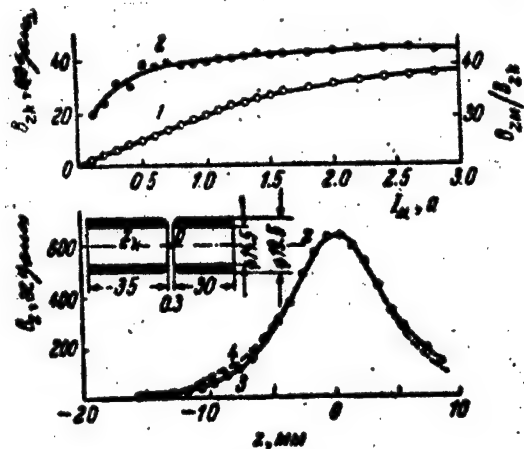but curve 4— the respective "ideal " field,calculated

Fig. 7.

per formula (2); curve 1-
$B_z(I_m)$ for z = z_k ;curve
2— B_{zm}/B_{zk} with relation
to I_m. As may be seen,the
field ratio B_{zm}/B_{zk} is
somewhat larger than need-
ed (calculated beam com-
pression is 34)

Fig. 8 shows a num-
ber of oscillograms of cur-
rent density distribution in the same place -at the mini-
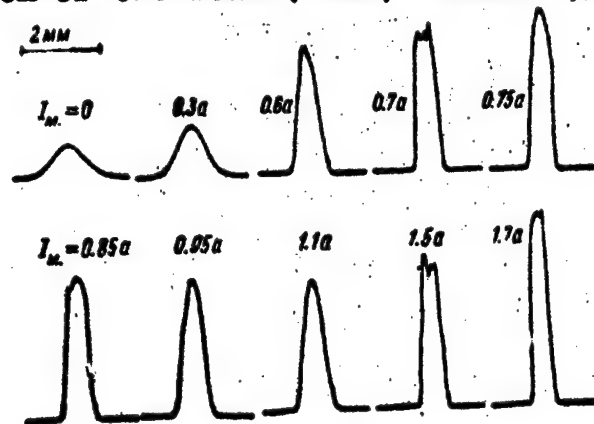mum crossection of the beam (z=0). These curves differ

Fig. 8.

only through the currents of the electromagnet,the magni-
tude of which is given at each curve. Measurements were
made at an anode voltage of 200 v and 1.1 ma current. The
scale of the curves per r is shown above curves 1 .

The oscillograms of Fig. 8 illustrate very clearly the influence of the magnetic field on the quality of the formed beam. The first curve shows the density distribution of current in absence of a magnetic field. The beam has indistinct boundaries and a low current density, due to the initial thermal velocities. With the increase of the magnetic field the current density increases, but the beam diameter decreases. The beam boundaries become distinct, and the current density distribution approaches the Π-type (for $I_m = 0.7-0.8$ a). With further increase of the magnetic field the focusing disappears and the beam again loses the distinct borderlines (curve $I_m = 1.1$ a), although the current density in the middle of the beam decreases only insignificantly. With a further increase of the magnetic field the focusing appears again   (for $I_m = 1.7$ a) etc.

Some peculiarities of the shown curves should be mentioned. With approach to the focusing point the current density distribution acquires an uneven, nicked shape with peaks at the edges (for instance, $I_m = 0.7$ a), but with further increase of I_m the current density in the center of the beam becomes higher than at the periphery (curves $I_m = 0.75 - 0.85$ a) . This can be explained by the difference of thermal electron trajectories, leaving the ca-

thode at the edges, and the central electrons, which is not considered in the theory. One can assume that first the peripheral electrons are focused, and after this the central electrons. This dependence of current density distribution on I_m, regularly repeated itself in the vicinity of all focusing points, independent of the anode voltage (measurements were made also for $U_a = 100v$ and $U_a = 400$ v).

It should be noted that between the focusing points the maximum current density drops little, takes place only a more shallow drop of it towards the edges. The deep depressions in the collector current curves on Fig.4 are obviously caused by a slight misalignment of the opening in the target with respect to the axis of the beam , otherwise the collector current would change only very slightly after the first maximum.
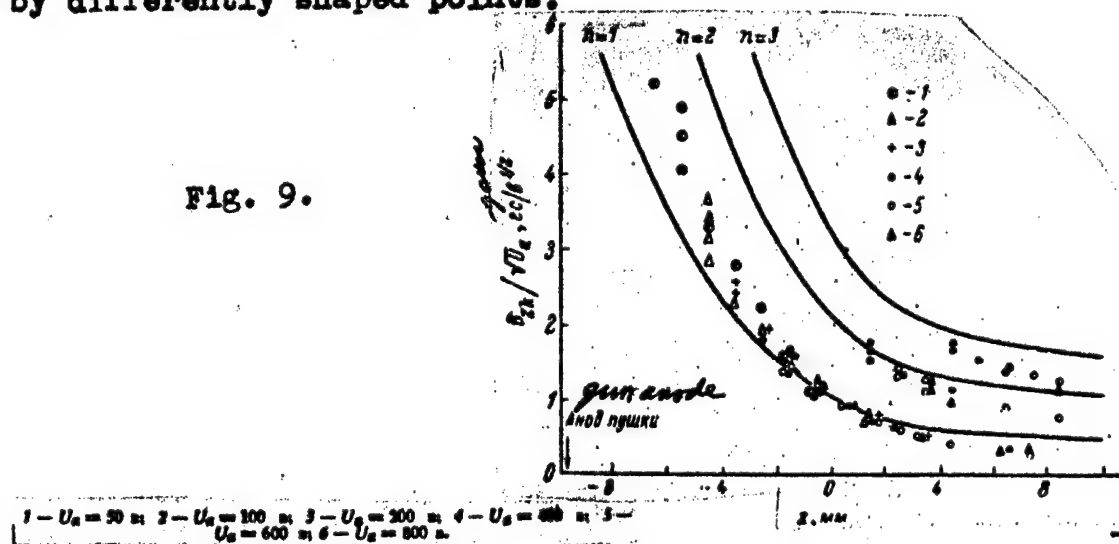
It should be remembered that since at the focusing point an image of the cathode is obtained, the shape of the current density distribution curve at this spot is determined by the current density distribution over the cathode surface. This seems to explain the slight asymmetry and uneven shape of some curves and also the doubling of the maxima on Fig. 5.

Besides the described ones a great number of curves were drawn

of current density at different distances from the gun, and accelerating voltages from 50 to 800 v at different magnitudes of the magnetic field. With increase of the distance from the gun anode the current density curves become sharper, and the picture described above repeats itself: the outer electrons are being focused first (the curves are similar to $I_m = 0.7$ a of Fig. 8), and afterwards the inner electrons (as for $I_m = 0.75-0.85$ a. With decrease of voltage and increase of the field the intervals on axis z between the focusing points become shorter.

Based on the obtained oscillograms (approx.1500 curves) the condition (1) was examined, as far as the dependence on the regime and the axial distance z is concerned. The focusing points calculated per (3) are shown on Fig. 9 in full lines, but the experimentally determined - by differently shaped points.

Fig. 9.

The comparison shows that condition (1) agrees satisfactorily with experimental data. The points shown on Fig. 9 correspond to the assumed focusing of the outer electrons (curves are analogous to $I_m = 0.7$ a Fig. 8), but for the inner electrons the points would be a little to the right.

An apparatus constructed per Fig. 6 with another Pierce gun ($\bar{r}_k / \bar{r}_a = 2.5$) permitted to obtain a beam with sharp boundaries at a compression of 125 ($U_a = 600v$, I $= 5.9$ ma)

Conclusions.

The data given above prove that the characteristics of an electrostatic forming system can be considerably improved if it is located in a magnetic field with force lines directed along the electron trajectories. It is possible to increase considerably the current density in the beam and to obtain distinct boundaries of it, which is desirable, especially in super-high-frequency equipment. Such an electron beam, stabilized by the magnetic field , can be used for example in combination with a magnetic focusing system in traveling-wave tubes , backward-wave tubes, etc. But as shown by calculations, a considerable increase of the focusing field, as compared with a gun without a magnetic field, is required,-by approx. 2.5 to

4 times ,if a good outline of the beam at the entrance
to the focusing field is required. It seems that the app-
lication of this system in equipment with short inter -
action space - klystrons,retarding field oscillators,etc.
In this case it is obviously possible to improve the
parameters of the equipment,as compared with the used
electrostatic forming systems. For producing the stabi
lizing field only a small number of ampere-turns is re-
quired,and the configuration of the magnetic system per-
mits the use of permanent magnets of satisfactory light
weight.

BIBLIOGRAPHY .

/1/ Yu.V.Troitskiy,Zh T F,XXX,25,1960.- /2/C.C.Cutler ,
M.E.Hines,Proc.IRE,43,307,1955.- /3/ W.E.Danielson ,J.L.
Rosenfeld,J.A.Saloom,Bell Syst.Techn.J.,35,375,1956.-
/4/ A.Szabo, Trans.IRE,ED-5,183,1958.-/5/ J.R.Pierce ,
Teoriya i raschet elektronnykh puchkov (Theory and Calcu-
lation of Electron Beams),pg 179-190,Sov.Radio,1956.-
/6/ A.Rustergolts,Elektronnaya optika (Electron Optics),
pg 52-61, I L ,1952.-/7/ K.Pöschl,W.Veith,Arch.Elektr.
Uebertr.,12,45,1958.- /8/ Yu.V.Troitskiy,V.I.Vazhenin ,
Izv.Sib.Otdel.AN SSSR,No 8,17,1959.

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